



DRAFT INTERNATIONAL STANDARD ISO/DIS 10110-5

ISO/TC 30  
TC 172/SC 1  
Voting begins on

Secretariat  
DIN  
Voting terminates on

INTERNATIONAL ORGANIZATION FOR STANDARDIZATION - МЕЖДУНАРОДНАЯ ОРГАНИЗАЦИЯ ПО СТАНДАРТИЗАЦИИ - ORGANISATION INTERNATIONALE DE NORMALISATION

### Optics and optical instruments - Preparation of drawings for optical elements and systems - Part 5: Surface form tolerances

DC-981123005

THIS DOCUMENT IS A DRAFT CIRCULATED FOR COMMENT AND APPROVAL. IT IS THEREFORE SUBJECT TO CHANGE AND MAY NOT BE REFERRED TO AS AN INTERNATIONAL STANDARD UNTIL PUBLISHED AS SUCH.

IN ADDITION TO THEIR EVALUATION AS BEING ACCEPTABLE FOR INDUSTRIAL, TECHNOLOGICAL, COMMERCIAL AND USER PURPOSES, DRAFT INTERNATIONAL STANDARDS MAY ON OCCASION HAVE TO BE CONSIDERED IN THE LIGHT OF THEIR POTENTIAL TO BECOME STANDARDS TO WHICH REFERENCE MAY BE MADE IN NATIONAL REGULATIONS.

International Organization for Standardization

FORM B (ISO)

## 1 Scope

This International Standard applies to the presentation of design and functional requirements for optical elements in technical drawings used for manufacturing and inspection.

This part of the Standard gives rules for the indication of the tolerance for surface form. It is complemented by Annexes 1 through 3. Annexes 1 and 2 contain instructions for the determination of the amounts of different surface form deviation types for given optical surfaces. Annex 3 addresses the physical effects of rms surface deviations.

This part of the Standard applies to spherical surfaces; provided that footnotes 2) and 3) are respected, it may also be applied to aspheric surfaces. (It is to be noted that ISO/DIS 10110 Part 12 allows the surface form tolerance for aspheric surfaces to be specified without reference to this part of the Standard.)

## 2 Normative references

The following standards contain provisions which, through reference in this text, constitute provisions of this International Standard. At the time of publication, the editions indicated were valid. All standards are subject to revision, and parties to agreements based on this International Standard are encouraged to investigate the possibility of applying the most recent editions of the standards indicated below. Members of IEC and ISO maintain registers of currently valid International Standards.

- ISO/DIS 10110 Part 1 Optics and optical instruments - Preparation of drawings for optical elements and systems - Part 1: General
- ISO/DIS 10110 Part 10 Optics and optical instruments - Preparation of drawings for optical elements and systems - Part 10: Table representing data of a lens element
- ISO/DIS 10110 Part 11 Optics and optical instruments - Preparation of drawings for optical elements and systems - Part 11: Non-toleranced data
- ISO/DIS 10110 Part 12 Optics and optical instruments - Preparation of drawings for optical elements and systems - Part 12: Aspheric surfaces

## 3 Definitions

### 3.1 Surface form deviation

Surface form deviation is the difference between the optical surface under test and the nominal theoretical surface, measured perpendicular to the theoretical surface, which shall be nominally parallel to the surface under test. For testing



purposes, the desired theoretical surface may be represented by a test glass, interferometric reference surface, or other measuring device of sufficient accuracy.

### 3.2 Peak-to-valley (PV) difference and peak-to-valley (PV) value

The peak-to-valley (PV) difference between two surfaces is equal to the maximum distance minus the minimum distance between the surfaces. If one of the surfaces is a theoretical surface, it is possible that the surfaces cross, in which case the minimum distance between the surfaces is a negative number; the sign must be taken into account in computing the PV difference.

The PV value of a surface is the PV difference between the surface and the planar surface which best approximates it.

### 3.3 Fringe spacings, wavelength

The surface form deviation is to be specified in "fringe spacings". One fringe spacing is a distance equal to one-half the specified light wavelength.

Unless otherwise specified, the wavelength is that of the green spectral line of mercury, 546.07 nm.

Specifications may be converted from one reference wavelength to another using the formula:

$$(\text{No. of fringe spacings at } \lambda_2) = (\text{No. of fringe spacings at } \lambda_1) \frac{\lambda_1}{\lambda_2}$$

### 3.4 Types of surface form deviation

The surface form tolerances are indicated by specification of the maximum permissible amounts of sagitta error, irregularity, and rotationally symmetric irregularity. These quantities are defined below.

#### 3.4.1 Sagitta error

Sagitta error results from the test surface having a radius of curvature different from the specified radius.

In order to determine the sagitta error of a spherical surface, it is first necessary to determine the spherical surface which best approximates the surface under test. This approximating spherical surface is defined as that spherical surface for which the root-mean-square (rms) difference to the test surface is a

minimum.<sup>1)</sup> (The rms deviation is defined in Annex 1.) The value of the sagitta error is the peak-to-valley difference between the desired theoretical surface and the approximating spherical surface.<sup>2)</sup>

Annex 1 gives one method for determining the amount of sagitta error of a given surface using digital interferogram analysis techniques.

Methods by which the amount of sagitta error can be estimated using test glasses or visual interpretation of interferograms are given in Annex 2.

#### 3.4.2 Irregularity

The irregularity of a nominally spherical surface is a measure of its departure from sphericity.

The value of the irregularity of an optical surface is equal to the peak-to-valley difference between the optical surface under test and the approximating spherical surface (see subclause 3.4.1). The theoretical surface defined by the difference between these two surfaces will be referred to as the irregularity function.<sup>3)</sup>

Annex 1 gives one method for determining the amount of irregularity of a given surface using digital interferogram analysis techniques.

Methods by which the amount of irregularity can be estimated using test glasses or visual interpretation of interferograms are given in Annex 2.

#### 3.4.3 Rotationally symmetric irregularity

Surfaces which are rotationally symmetric, but do not have the desired shape, are said to have rotationally symmetric irregularity. This error is the rotationally symmetric part of the irregularity function (see subclause 3.4.2).

In order to determine the value of the rotationally symmetric irregularity, it is first necessary to determine the rotationally symmetric aspheric surface which best approximates the surface under test. This approximating surface is defined as that rotationally symmetric surface for which the rms difference to the irregu-

- 1) For non-circular test areas, the remarks given in subclause 3.6 apply.
- 2) If the desired theoretical surface is aspheric, it is necessary to determine the total interferometric error function, which is defined as the difference between the actual surface and the desired theoretical surface. The approximating spherical surface is defined as that spherical surface which best approximates the total interferometric error function. The sagitta error is then equal to the PV value of the approximating spherical surface.
- 3) If the desired theoretical surface is aspheric, then the irregularity function is defined as the difference between the total interferometric error function and the approximating spherical surface (see footnote 2)). The irregularity is the PV value of the irregularity function.



larity function is a minimum<sup>11</sup>. The value of the rotationally symmetric irregularity is the PV value of the approximating aspheric surface.

Because the rotationally symmetric irregularity is only a part of the total irregularity, its value cannot exceed that of the irregularity.

Annex 1 gives one method for determining the amount of rotationally symmetric irregularity of a given surface using digital interferogram analysis techniques.

Methods by which the amount of rotationally symmetric irregularity can be estimated using test glasses or visual interpretation of interferograms are given in Annex 2.

### 3.5 Types of root-mean-square (rms) residual deviation

Several types of rms residual deviation are defined below. Each type represents the root-mean-square value of the function remaining after the subtraction of certain specified surface deviation types. The amount of each type of deviation to be removed is that amount which minimises the rms residual deviation (The rms deviation is defined mathematically in Annex 1.)

The value of the rms residual deviation of a given optical surface cannot be determined visually, and digital techniques are therefore required.

#### 3.5.1 Total rms deviation, RMSt

The total rms deviation is defined as root-mean-square difference between the optical surface under test and the nominal theoretical spherical surface, without subtraction of any surface form deviation types.

Annex 1 gives method for the calculation of the value of RMSt for a given optical surface.

#### 3.5.2 Rms irregularity, RMSi

The rms irregularity is defined as the root-mean-square difference between the optical surface under test and the approximating spherical surface (see subclause 3.4.1). This is equivalent to the rms value of the function remaining after the spherical approximating surface defined in subclause 3.4.1 has been subtracted from the total surface form deviation.

Annex 1 gives one method for the calculation of the value of RMSi for a given optical surface.

### 3.5.3 Rms asymmetry, RMSa

The rms asymmetry is defined as the root-mean-square value of the difference between the optical surface under test and the approximating aspheric surface (see subclause 3.4.3). This is equivalent to the rms value of the function remaining after removal of sagitta error and non-spherical, rotationally symmetric error from the total surface form deviation.

Annex 1 gives method for the calculation of the value of RMSa for a given optical surface.

### 3.6 Non-circular test areas

For non-circular test areas, the peak-to-valley (PV) and root-mean-square (RMS) values given in subclause 3.4 are to be calculated within the actual test area only. It is important to note that for non-circular test areas, the spherical surface which minimizes the rms difference to the surface under test (subclause 3.4.1) is not the spherical part of an approximating surface which is aspheric. Also, the rotationally symmetric surface which minimizes the rms difference to the irregularity function (subclause 3.4.4) is not the rotationally symmetric part of an approximating surface which is not rotationally symmetric.

## 4 Specification of tolerances for surface form deviation

The maximum permissible values for sagitta error, irregularity, and rotationally symmetric irregularity are to be specified in units of fringe spacings.

If a specification is to be given for one or more rms deviation types, this is to be done in units of fringe spacings. It is to be noted that specification of a tolerance for an rms deviation type requires that the surface be tested with a digital interferometer.

It is not necessary that tolerances be specified for all types of surface deviations.

### 4.1 Indication

The surface form tolerance is indicated by a code number and the indications of the tolerances for sagitta error, irregularity, non-spherical rotationally symmetric error and rms deviation types, as appropriate.

The code number for surface form tolerance is 3/.



The indication shall have one of the three forms:

3/A(B/C),

or

3/A(B/C) RMSx < D (where (x) is one of the letters t, i, or a),

or

3/- RMSx < D (where (x) is one of the letters t, i, or a).

The quantity A is either:

1) the maximum permissible sagitta error expressed in fringe spacings, as defined in subclause 3.4.1<sup>4)</sup>,

or

2) a dash (-) indicating that the total radius of curvature tolerance is given in the radius of curvature dimension (not applicable for planar surfaces).

The quantity B is either:

1) the permissible PV value of irregularity, expressed in fringe spacings, as defined in subclause 3.4.2

or

2) a dash (-) indicating that no explicit irregularity tolerance is given.

The quantity C is the permissible rotationally symmetric irregularity expressed in fringe spacings, as defined in subclause 3.4.3. If no tolerance is given the divisor line (/) is replaced by the final parenthesis, i.e. 3/A(B).

If no tolerance is given for all three deviation types, then A, B, C, the divisor line (/) and the parenthesis are replaced by a single dash (-), i.e., 3/-.

The quantity D is the maximum permissible value of the rms quantity of the type specified by (x), where (x) is one of the letters t, i, or a. These deviations are defined in subclause 3.5. The specification of more than one type of rms deviation is allowed, see example 5.

The surface form tolerance indicated as described above applies to the optically effective area, except when the indication is to apply to a smaller test field for all possible positions within the optically effective area. In this case the diameter of the test field shall be appended to the tolerance indication as follows:

3/A(B/C), RMSx < D (all  $\phi$ ...),

(see example 3).

4) It is often the case that the tolerance for the sagitta error is calculated by converting only part of the tolerance shown against the radius of curvature tolerance into a tolerance for the sagitta error, according to subclause 4.3.

#### 4.2 Location

The indication shall be shown in connection with a leader to the surface to which it relates and will be associated with centering errors and surface imperfections. An example of such indication is given in the annex to ISO/DIS 10110 part 1.

Alternatively, for lens elements, the indication may be given in a table according to ISO/DIS 10110 part 10.

If two or more optical elements are to be cemented (or optically contacted), the surface form tolerances given for the individual elements apply also for the surfaces of the optical sub-assembly, i.e. after cementing (or optically contacting), unless otherwise specified. See ISO/DIS 10110 part 1: General, subclause 4.8.3.

#### 4.3 Relationship between sagitta error tolerance and radius of curvature tolerance

To determine the number of fringe spacings corresponding to a dimensional radius of curvature tolerance, the following formula may be used, provided that the ratio  $\frac{\Delta R}{R}$  is small:

$$N = \frac{2 \Delta R}{\lambda} \left\{ 1 - \sqrt{1 - \left[ \frac{\sigma}{2R} \right]^2} \right\}$$

If the ratio  $\frac{\sigma}{R}$  is small, this formula may be approximated by

$$N = \left[ \frac{\sigma}{2R} \right]^2 \cdot \frac{\Delta R}{\lambda}$$

where:

- N is the maximum permissible number of fringe spacings,
- R is the radius of curvature,
- $\Delta R$  is the dimensional radius of curvature tolerance,
- $\sigma$  is the diameter of the test area, and
- $\lambda$  is the wavelength (normally, 546,07 nm).

#### 5 Examples of tolerance indications

Example 1: 3/3(1).

The tolerance for sagitta error is 3 fringe spacings. The irregularity may not exceed 1 fringe spacing.



Example 2: 3/5(-) RMSi < 0,05.

The tolerance for the sagitta error is 5 fringe spacings. No specific tolerance is given for irregularity or rotationally symmetric irregularity, but the rms value of the irregularity may not exceed 0,05 fringe spacings.

Example 3: 3/3(1/0,5), (all  $\phi$  20).

The tolerance for the sagitta error is 3 fringe spacings. The total irregularity may not exceed 1 fringe spacing. The rotationally symmetric irregularity may not exceed 0,5 fringe spacings. This tolerance applies for all possible test fields of diameter 20 within the total test area.

Example 4: 3/-(1).

No specific tolerance for the sagitta error is given; the tolerance on the radius of curvature is to be taken from the radius of curvature indication<sup>5)</sup>. The total irregularity may not exceed 1 fringe spacing.

Example 5: 3/- RMSi < 0,07, RMSa < 0,035.

No specific tolerance for the sagitta error, irregularity, or rotationally symmetric irregularity is given; the tolerance on the radius of curvature is to be taken from the radius of curvature indication<sup>5)</sup>; however when the surface is compared to the desired theoretical spherical surface, the total rms deviation must be less than 0,07 fringe spacings, and the rms asymmetry less than 0,035 fringe spacings.

5) Table 1 of ISO/DIS 10110 part 11 applies, if no tolerance on the radius of curvature is specified.

## ANNEX 1

## Digital interferogram analysis

- This annex is informative in nature; it provides a method for the analysis of surfaces which can be described in terms of polynomials.
- The contents of this annex are important for users of digital interferometers as well as for developers of software for interferometry.
- Examples of surfaces, to which this method does not apply are surfaces having interferometric error functions which are cone-shaped and surfaces with spatially localised errors.

## A1.1 General

The amounts of the various types of surface form deviation are determined through a process of successive fitting and removal of deviation types; at each stage, the removal of one type of surface form deviation exposes the next type of deviation.

The procedure by which a function of a certain type which "best fits" a certain original function is the well-known method of least squares, which minimises the rms error between the original function and the approximation to it. The rms value of a function is defined in subclause A1.4.

## A1.1.1 Effective reference surface

When testing curved surfaces interferometrically, the surface under test is compared with a reference wavefront. The resulting fringe pattern represents the difference between the surface under test and the projection of the reference wavefront onto the surface under test. This projected wavefront will be referred to as the effective reference surface.

The apparent surface figure deviations as measured by the interferometer (including the relative tilt between the surface under test and the interferometric reference surface) will be referred to as the wavefront error function  $W(r, \theta)$ .

## A1.1.2 Coordinate system

The surface of the optical surface under test is described in polar coordinates by the variables  $r$  and  $\theta$ ; the origin of the coordinate system is the centre of the test area, and  $r$  is normalised to one at the edge of the test area. For non-circular test areas, the "centre" of the test area refers to its centroid, and the radius of the test area refers to the distance from the center to the most distant point. The parameter  $r$  ranges therefore between zero and one.



Various approximations to the surface are represented as linear combinations of the polynomials - commonly called Zernike polynomials -  $Z_0(r,\theta), Z_1(r,\theta)$ . These combinations are given by corresponding coefficients  $C_0, C_1, \dots$

## A1.2 Procedure

The procedure for finding the amounts of the various surface form deviations is given in subclauses A1.2.1 through A1.2.7. Although this procedure is described in terms of the Zernike polynomials (subclause A1.3), any mathematically equivalent procedure, based on another set of function may be used; however, the deviations must be determined and subtracted in the order specified here.

### A1.2.1 The total interferometric error

To the measured wavefront error function  $W(r,\theta)$ , the best fitting plane  $P(r,\theta) = C_0Z_0 + C_1Z_1 + C_2Z_2$  is found by the least squares procedure. The total interferometric error function (TIE) is found by subtracting the best fitting plane from the measured wavefront error:

$$TIE(r,\theta) = W(r,\theta) - P(r,\theta)$$

### A1.2.2 The total rms deviation RMSt

If the radius of the effective reference surface is equal to the radius of the desired theoretical surface, then the total rms deviation (RMSt, subclause 3.5.1) is equal to the rms value of the total interferometric error function  $TIE(r,\theta)$ . The quantity RMSt can not be directly determined, if effective reference surface and theoretical surface have different radii.

### A1.2.3 The approximating spherical surface and the sagitta error

Usually, the effective reference surface closely matches the surface under test. In such a case, the difference between these two spherical surfaces can be approximated by fitting a second-order function of the radial variable  $r$  to the total interferometric error function.

$$\text{Approximating sphere} = C_3Z_3$$

The sagitta error (subclause 3.4.1) is given by the expression:

$$\text{Sagitta error} = \sqrt{2} C_3 r^2 \quad \text{(A1.1)}$$

If the radius of the effective reference wavefront does not correspond to that of the nominal theoretical spherical surface, then the sagitta difference between these two spheres must be added to the interferometrically determined sagitta

error, determined as above. (If the radius of the effective reference surface is unknown, then the sagitta error of the surface cannot be determined.)

If the diameter of the test area is not much smaller than its radius of curvature, the difference between the two spheres will contain higher order terms. In order to distinguish between these and terms which represent rotationally symmetric irregularity, a function which more closely represents the difference between two spheres must be used in place of  $Z_3$ .

#### A1.2.4 The irregularity function

The irregularity function  $IRR(r,\theta)$  is the difference between the total interferometric error function  $TIE(r,\theta)$  and the approximating sphere. This corresponds to the function remaining after the sagitta error has been removed from the wavefront.

$$IRR(r,\theta) = TIE(r,\theta) - C_3 Z_3 .$$

#### A1.2.5 The irregularity and the rms irregularity, RMSi

The rms irregularity RMSi (subclause 3.5.2) is equal to the rms value of the irregularity function. The irregularity (subclause 3.4.2) is equal to the peak-to-valley value of the irregularity function. However, some form of smoothing (e.g. convolution or replacement of the function with a polynomial of sufficient order) is usually required to remove the effects of isolated surface defects (scratches, etc.), scattering of light from dust particles, and measurement "noise" which are not part of the surface form deviation.

#### A1.2.6 The approximating spherical surface and the rotationally symmetric irregularity

The approximating aspheric surface  $AAS(r,\theta)$  is obtained by a least squares fit of a series of rotationally symmetric Zernike polynomials to the irregularity function  $IRR(r,\theta)$ :

$$AAS(r,\theta) = C_8 Z_8 + C_{16} Z_{16} + C_{24} Z_{24} + C_{35} Z_{35} + \dots$$

In most cases, the approximation is sufficiently accurate using the four terms listed above. Higher order terms may be used if necessary.

The rotationally symmetric irregularity (subclause 3.4.3) is equal to the peak-to-valley value of the approximating aspheric surface  $AAS(r,\theta)$ . This may be determined in practice by calculating the value of  $AAS(r,\theta)$  at discrete points located on a sufficiently fine grid and taking the difference between the highest and lowest values.



### A1.2.7 The rms asymmetry RMSa

The approximating aspheric surface  $AAS(r, \theta)$  is subtracted from the irregularity function  $(IRR(r, \theta))$ ; the rms asymmetry (subclause 3.5.3) is the rms value of the function which remains.

### A1.2.8 Provisions for aspheric surfaces

The procedure for the evaluation of interferograms resulting from aspheric surfaces is essentially the same as described above; however the footnotes 2 and 3 of subclause 3.4 must be respected.

### A1.3 The Zernike polynomials

The set of polynomials identified by Zernike and Nijboer (see ref. [A1.1]) as being orthogonal in the sense of rms integration over a circular area, are commonly used for interferogram analysis. For circular test areas, the analysis may be simplified by the orthogonality properties of these polynomials. For non-circular pupils, these polynomials are no longer orthogonal and no longer offer any advantage over other sets of functions; however, they may still be used, provided that the analysis techniques given in subclause A1.2 are used.

$$Z_0(r, \theta) = 1$$

$$Z_1(r, \theta) = r \cdot \cos \theta$$

$$Z_2(r, \theta) = r \cdot \sin \theta$$

$$Z_3(r, \theta) = 2r^2 - 1$$

$$Z_4(r, \theta) = r^2 \cdot \cos 2\theta$$

$$Z_5(r, \theta) = r^2 \cdot \sin 2\theta$$

$$Z_6(r, \theta) = (3r^2 - 2) \cdot r \cdot \cos \theta$$

$$Z_7(r, \theta) = (3r^2 - 2) \cdot r \cdot \sin \theta$$

$$Z_8(r, \theta) = 6r^4 - 6r^2 + 1$$

$$Z_9(r, \theta) = r^3 \cdot \cos 3\theta$$

$$Z_{10}(r, \theta) = r^3 \cdot \sin 3\theta$$

$$Z_{11}(r, \theta) = (4r^2 - 3) \cdot r^2 \cdot \cos 2\theta$$

$$Z_{12}(r, \theta) = (4r^2 - 3) \cdot r^2 \cdot \sin 2\theta$$

$$Z_{13}(r, \theta) = (10r^4 - 12r^2 + 3) \cdot r \cdot \cos \theta$$

$$Z_{14}(r, \theta) = (10r^4 - 12r^2 + 3) \cdot r \cdot \sin \theta$$

$$Z_{15}(r, \theta) = 20r^6 - 30r^4 + 12r^2 - 1$$

$$Z_{16}(r, \theta) = r^4 \cdot \cos 4\theta$$

$$Z_{17}(r, \theta) = r^4 \cdot \sin 4\theta$$

$$\begin{aligned}
Z_{18}(r, \theta) &= (5 r^2 - 4) r^3 \cdot \cos 3\theta \\
Z_{19}(r, \theta) &= (5 r^2 - 4) r^3 \cdot \sin 3\theta \\
Z_{20}(r, \theta) &= (15 r^4 - 20 r^2 + 6) r^2 \cdot \cos 2\theta \\
Z_{21}(r, \theta) &= (15 r^4 - 20 r^2 + 6) r^2 \cdot \sin 2\theta \\
Z_{22}(r, \theta) &= (35 r^6 - 60 r^4 + 30 r^2 - 4) r \cdot \cos \theta \\
Z_{23}(r, \theta) &= (35 r^6 - 60 r^4 + 30 r^2 - 4) r \cdot \sin \theta \\
Z_{24}(r, \theta) &= 70 r^8 - 140 r^6 + 90 r^4 - 20 r^2 + 1 \\
Z_{25}(r, \theta) &= r^6 \cdot \cos 5\theta \\
Z_{26}(r, \theta) &= r^6 \cdot \sin 5\theta \\
Z_{27}(r, \theta) &= (6 r^2 - 5) r^4 \cdot \cos 4\theta \\
Z_{28}(r, \theta) &= (6 r^2 - 5) r^4 \cdot \sin 4\theta \\
Z_{29}(r, \theta) &= (21 r^4 - 30 r^2 + 10) r^3 \cdot \cos 3\theta \\
Z_{30}(r, \theta) &= (21 r^4 - 30 r^2 + 10) r^3 \cdot \sin 3\theta \\
Z_{31}(r, \theta) &= (56 r^6 - 105 r^4 + 60 r^2 - 10) r^2 \cdot \cos 2\theta \\
Z_{32}(r, \theta) &= (56 r^6 - 105 r^4 + 60 r^2 - 10) r^2 \cdot \sin 2\theta \\
Z_{33}(r, \theta) &= (126 r^8 - 280 r^6 + 210 r^4 - 60 r^2 + 5) r \cdot \cos \theta \\
Z_{34}(r, \theta) &= (126 r^8 - 280 r^6 + 210 r^4 - 60 r^2 + 5) r \cdot \sin \theta \\
Z_{35}(r, \theta) &= 252 r^{10} - 630 r^8 + 560 r^6 - 210 r^4 + 30 r^2 - 1
\end{aligned}$$

#### A1.4 Root-mean-squares (rms) value of a function

The root-mean-square value of a function  $f$  of two variables  $x$  and  $y$  over a given area  $A$  is given by the integral expression:

$$\text{RMS value} = \left[ \frac{\int_A [f(x, y)]^2 dA}{\int_A dA} \right]^{\frac{1}{2}}$$

This integral may be approximated by a corresponding summation, provided that a sufficient number of data is used.

#### References:

- [A1.1] A discussion of these polynomials and their properties is given in M. Born and E. Wolf, Principles of Optics, Pergamon press, Elmsford, New York.



## ANNEX 2

## Visual interferogram analysis

This annex is informative in nature; it is intended as an aid to understanding Part 5 of ISO/DIS 10110. The annex is useful for the interpretation of interferograms (including fringe patterns seen when using test glasses), but the guidelines given below for the estimation of the amounts of the various surface form deviations do not serve to define these surface form deviation types.

## A2.1 General

The main purpose of this annex is to demonstrate the visual appearance of the different form errors; for ease of readability, only the case of nominally spherical test surfaces is described. Aspheric surfaces may also be visually evaluated, provided that footnotes 2) and 3) of subclause 3.4 are respected.

This annex deals exclusively with the following types of surface form deviation: sagitta error, irregularity, and rotationally symmetric irregularity. The rms residual deviation types (described in subclause 3.5) cannot be determined by visual inspection.

Subclauses A2.2 and A2.3 describe the analysis of circular test areas. Special considerations for non-circular test areas are given in subclause A2.2.4.

## A2.1.1 Interferometric tilt

Two methods are used for estimating the amounts of sagitta error and irregularity, depending on whether or not the relative tilt between the reference surface and the surface under test can be adjusted. The method without tilt is applied chiefly when using test glasses and when the surface form deviation is large. The method employing tilt is generally more accurate.

## A2.1.2 Effective reference surface

The sagitta error can only be determined if the radius of curvature of the effective reference surface is known. When using test glasses, this is equal to the radius of the test glass itself. When testing curved surfaces with a non-contact interferometer, the apparent sagitta error depends on the distance between the test surface and the reference surface. The effective reference surface is the projection of the reference surface onto the surface under test. Often, the radius of the effective reference surface is unknown, and the sagitta error cannot be determined; however, the irregularity can still be determined.

The determination of the sagitta error is the simplest when the radius of curvature of the effective reference surface is equal to that of the nominal theoretical

surface. In the following, it is assumed that this is the case. (If this is not the case, then the difference between the sagittas of the nominal theoretical surface and the effective reference surface must be added - taking the sign into account - to the sagitta error determined as described below. For this reason it is necessary to determine whether the surface is concave or convex with respect to the interferometric reference surface.)

## A2.2 Estimation of sagitta error and irregularity

Usually, the surface form deviation is dominated by sagitta error and/or by a kind of asymmetry in the sagitta error. In the case of asymmetry, cross-sections of the surface in different directions show different amounts of sagitta error. Other kinds of surface irregularity are possible; the estimation of their amounts is more difficult. The estimation of the amounts of sagitta error and irregularity for the commonly occurring cases is described in subclauses A2.2.1 and A2.2.2, and a more general procedure for unusual types of irregularity is described in A2.2.3. Reference [A2.1] contains a more thorough discussion of interferogram analysis.

### A2.2.1 Analysis of interferograms without tilt

In the absence of all other types of deviation, sagitta error causes an interference pattern having concentric, circular fringes. The radii of the fringes increase with the square root of the fringe number, counting from the centre of the fringe pattern.

If small amounts of asymmetric deviations are present, the circles distort into ovals, as shown in fig. A2.1. If the surface under test is concave with respect to the reference surface, then the fringes will move towards the centre of the fringe pattern. If the reverse is true, then the surface under test is convex with respect to the reference surface.

If large amounts of asymmetric deviations are present, the oval fringes may be broken into approximately hyperbolic fringes, as shown in fig. A2.2. In this case, when the surface under test is moved slightly towards the interferometric reference surface, some of the fringes will move towards the centre of the fringe pattern and some will move away from the centre.

To estimate the amount of sagitta error and irregularity in a test surface, let  $m$  and  $m'$  be the numbers of fringe spacings seen in the fringe pattern, counted from the centre to the edge, in the direction which give the largest and smallest numbers of fringes.<sup>1)</sup> In the case of oval fringes, the sagitta error is given by the average of  $m$  and  $m'$ , that is:

$$\text{Sagitta error (oval fringes)} = \frac{m + m'}{2} \lambda \quad \text{(A2.1)}$$

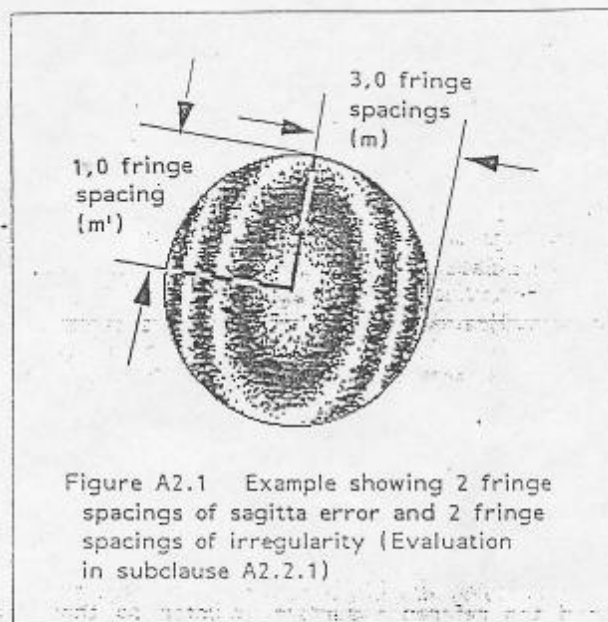
1) Usually, these two directions are oriented at 90 degree to one another, but this need not be the case.



In this case the irregularity is equal to the absolute value of the difference of the fringe counts  $m$  and  $m'$ :

$$\text{Irregularity (oval fringes)} = |m - m'| \quad (\text{A2.2})$$

In fig. A2.1, the values of  $m$  and  $m'$  are 1 and 3 fringe spacings; therefore, the sagitta error is  $(3+1)/2 = 2$  fringe spacings and the irregularity is  $|3-1| = 2$  fringe spacings.

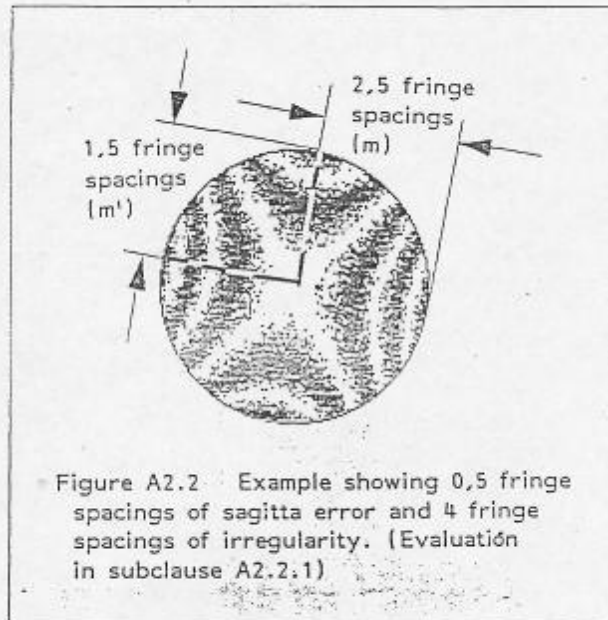


In the case of hyperbolic fringes, the sagitta error is equal to:

$$\text{Sagitta error (hyperbolic fringes)} = \frac{|m - m'|}{2} \quad (\text{A2.3})$$

and the irregularity is given by:

$$\text{Irregularity (hyperbolic fringes)} = m + m' \quad (\text{A2.4})$$



In fig. A2.2, the values of  $m$  and  $m'$  are 2,5 and 1,5 fringe spacings, respectively, so the sagitta error is  $|2,5 - 1,5|/2 = 0,5$  fringe spacings, and the irregularity is  $2,5 + 1,5 = 4$  fringe spacings.

#### A2.2.2 Analysis of fringe pattern with tilt

This method requires the fringes to be observed twice, with the tilt between the surface under test and the reference surface adjusted so that the fringes are oriented in two different directions. It is necessary that the adjustment of the tilt be made without changing the distance between the surface under test and the reference surface.

When the surface under test is tilted with respect to the reference surface, the fringes appear as in fig. A2.3. If only sagitta error is present, then the fringes appear as parts of concentric circles. The radii of the fringes increase with the fringe number, counting from the apparent centre of the fringe pattern. If other surface deviation types are also present, the fringes are not parts of concentric circles.



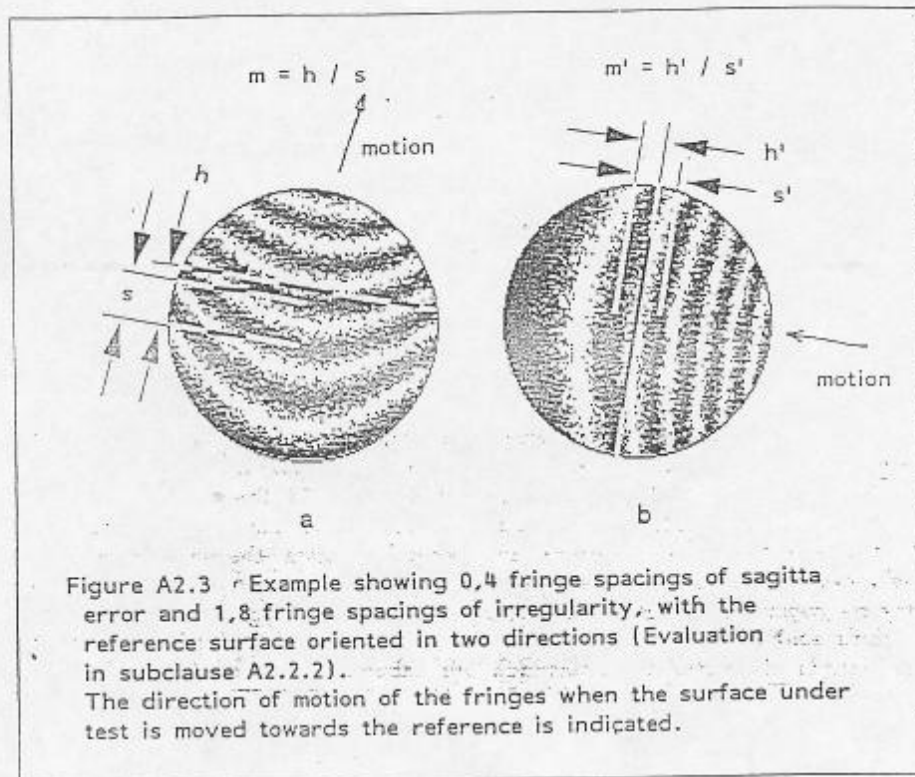


Figure A2.3 Example showing 0,4 fringe spacings of sagitta error and 1,8 fringe spacings of irregularity, with the reference surface oriented in two directions (Evaluation in subclause A2.2.2). The direction of motion of the fringes when the surface under test is moved towards the reference is indicated.

To estimate the sagitta error and the irregularity, it is necessary to estimate the curvature of the surface in the cross-section parallel to the fringes, for the two directions of tilt which give the maximum and minimum amounts of curvature (fig. A2.3(a) and (b)). In each case, the curvature  $m$  is equal to the curvature  $h$  of the fringe closest to the centre of the interferogram, divided by the spacing  $s$  of the fringes, which is also measured as close as possible to the centre of the test area.

In addition, it is necessary to note (for both directions of the tilt) the direction of motion of the fringes when the surface under test is moved slightly towards the reference surface.

If the fringes in both cases move towards the apparent centre of curvature of the fringes, or if the fringes in both cases move away from the apparent centre, then the sagitta error exceeds the irregularity, and Eqs.(A2.1) and (A2.2) shall be used to estimate the sagitta error and the irregularity, respectively. In this case, if the motion of the fringes is towards the apparent fringe centre, then the surface under test is concave with respect to the reference surface. Otherwise, the surface is convex with respect to the reference surface.

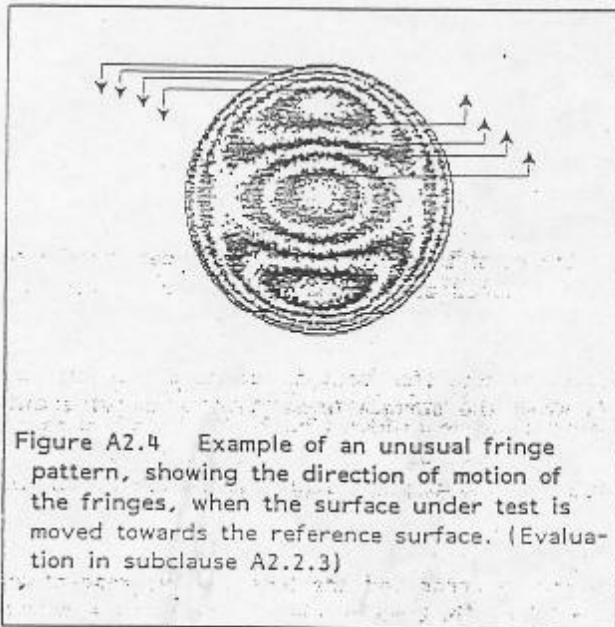
If one set of fringes moves towards its apparent centre, and the other fringe pattern moves away from its apparent centre, then the irregularity exceeds the sagitta error, and Eqs.(A2.3) and (A2.4) shall be used to estimate the amounts of sagitta error and irregularity. If the set of fringes with the larger curvature ( $m$  or  $m'$ ) moves towards its apparent centre, then the surface is concave with

respect to the reference surface; otherwise, it is convex with respect to the reference surface.

In fig. A2.3(a), the curvature  $h$  is approximately 1,3 times the fringe spacing  $s$ , so  $m = 1,3$ . In fig. A2.3(b), the curvature  $h'$  is one-half the fringe spacing  $s'$ , so  $m' = 0,5$ . The directions of motion shown in the figure indicate that Eqs.(A2.3) and (A2.4) are to be applied, from which the values stated in the figure caption result.

#### A2.2.3 Unusual forms of irregularity

It is possible that the form deviation of a surface be a maximum at some point inside the test area, rather than at the edge. When testing surfaces with no tilt between the interferometric reference surface and the surface under test, this leads to closed fringes which may not be concentric with the centre of the test area, as shown in fig. A2.4. In cases such as this, it is necessary to note which fringes move away from the centre and which towards the centre when the surface is moved towards the reference surface. Those which move towards the centre may be regarded as "positive", and the others as "negative".



The sagitta error is determined, according to Eq.(A2.1), where  $m$  and  $m'$  represent the cumulative numbers of fringes measured in two representative directions. In the vertical cross-section of fig. A2.4, there are 4 fringe intervals in the negative direction, followed by 4 fringe intervals in the positive direction, giving a value of zero for  $m$ . In the horizontal direction, there are 2 negative and 2



positive fringe intervals, again giving  $m' = \text{zero}$ . According to Eq.(A2.1), the sagitta error is:  $(0 + 0)/2 = 0$ .

The irregularity is determined by finding the highest and lowest departures from the theoretical expected fringe pattern, which is that the fringes are concentric circles with radii increasing as the square root of the fringe number. The irregularity is the sum of the absolute values of the highest and lowest departures from the pattern, measured in fringe intervals. For the pattern of fig. A2.4, the sagitta error is zero, so the theoretical expected fringe pattern has no fringes. The lowest departure from this is  $-4$  fringe intervals, at the centres of the two outer oval patterns, and the highest departure from this is zero. Therefore, the irregularity is  $|0| + |-4| = 4$  fringe intervals.

The analysis of fringe patterns is treated more fully in many textbooks, such as Reference [A2.1].

#### A2.2.4 Non-circular test areas

According to the definition of sagitta error in subclause 4.3.1 the sagitta error is based on the spherical surface which best approximates the surface under test. When using visual analysis methods, the approximating sphere is chosen so that the irregularity (which is the difference between the approximating sphere and the surface under test) is evenly distributed around the boundary of the test area. This requires that sagitta error and irregularity be evaluated by a method similar to that described in subclause A2.2, except that the calculations take into account the dimensions of the test area in the two cross-sections in which  $m$  and  $m'$  are measured.

For non-circular test areas, the "centre" of the test area refers to its centroid ("centre-of-gravity"), and its "radius" is equal to the distance from the centre to the most distant point in the test area.

The cross-sectional curvatures  $m$  and  $m'$  are determined in the same way as in subclause A2.2, using the description of the case with or without tilt, as appropriate. The directions along which  $m$  and  $m'$  are determined are given by the symmetry of the surface form error; these directions are not necessarily related to the shape of the test area.

Let  $m$  and  $m'$  be the cross-sectional curvatures in the two directions of symmetry, from the centre to the edge of the test area, as shown in fig. A2.5. Let  $a$  be the distance from the centre to the edge of the test area in the direction along which the curvature  $m$  is measured. Similarly, let  $b$  be the distance along which the curvature  $m'$  is measured. Let  $R$  be the radius of the test area, as defined above.

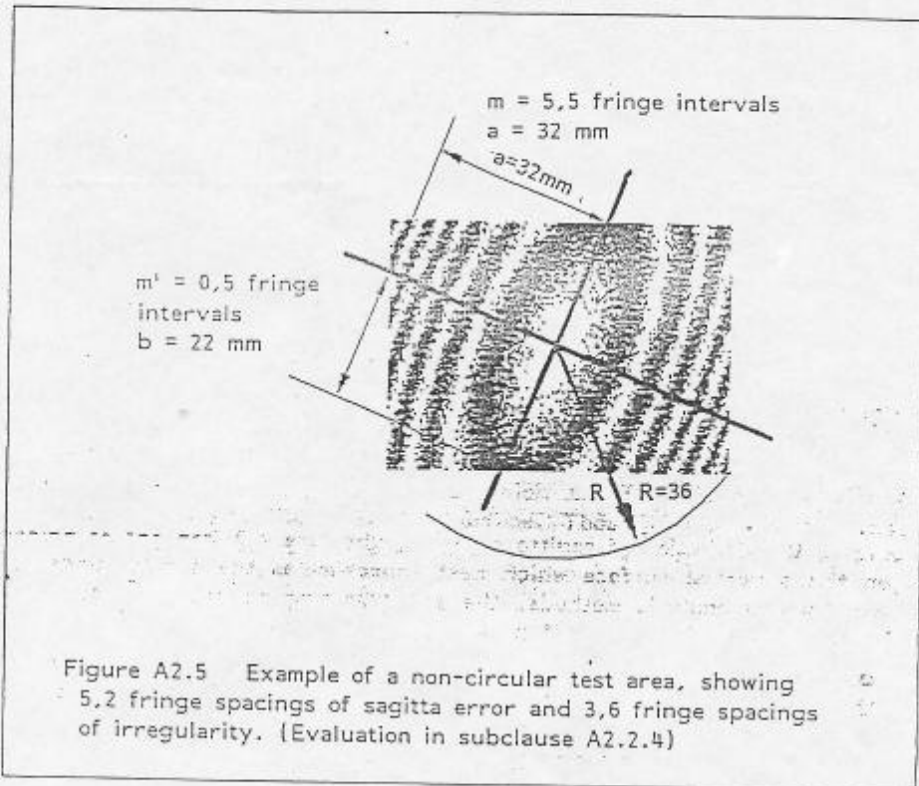


Figure A2.5 Example of a non-circular test area, showing 5.2 fringe spacings of sagitta error and 3.6 fringe spacings of irregularity. (Evaluation in subclause A2.2.4)

In the case of elliptical fringes, the sagitta error and the irregularity are determined by:

$$\text{Sagitta error (oval fringes)} = \frac{R^2 (m + m')}{a^2 + b^2} \quad (\text{A2.5})$$

$$\text{Irregularity (oval fringes)} = \left| \frac{2 R^2 (a^2 m' - b^2 m)}{a^2 (a^2 + b^2)} \right| \quad (\text{A2.6})$$

In fig. A2.5, the values of  $m$  and  $m'$  are 5.5 and 0.5 fringe spacings, measured over distances of 32 and 22 mm respectively. The radius of the test area is 36 mm. The sagitta error is found from Eq.(A2.5) to be 5.2 fringe spacings and the irregularity is found from Eq.(A2.6) to be 3.6 fringe spacings.



In the case of hyperbolic fringes, the sagitta error and irregularity are to be found by:

$$\text{Sagitta error (hyperbolic fringes)} = \frac{R^2 (m - m')}{a^2 + b^2} \quad (\text{A2.7})$$

$$\text{Irregularity (hyperbolic fringes)} = \left| \frac{2 R^2 (a^2 m' + b^2 m)}{a^2 (a^2 + b^2)} \right| \quad (\text{A2.8})$$

If there is tilt between the interferometric reference surface and the surface under test, then it is necessary to note (for both directions of the tilt) the direction of motion of the fringes when the surface under test is moved slightly towards reference surface.

If the fringes in both cases move towards the apparent centre of the fringe pattern, or if the fringes in both cases move away from the apparent centre of the fringe pattern, then the sagitta error exceeds the irregularity, and Eqs.(A2.5) and (A2.6) shall be used to estimate the sagitta error and the irregularity.

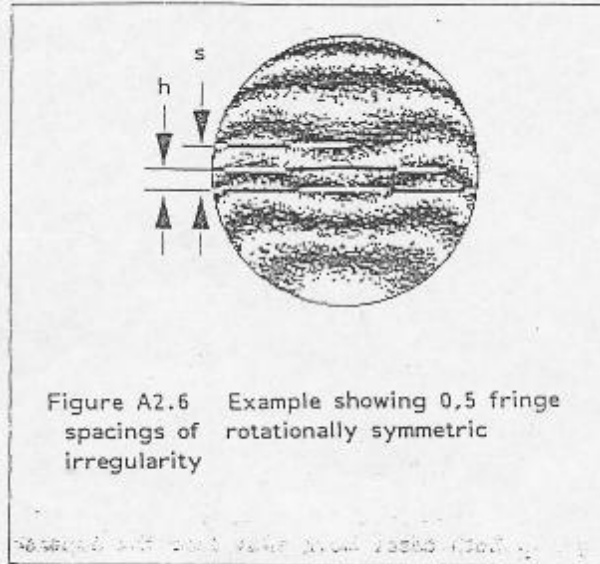
If one set of fringes moves towards its apparent centre, and the other fringe pattern moves away from its apparent centre, then the irregularity exceeds the sagitta error, and Eqs.(A2.7) and (A2.8) shall be used to estimate the amounts of sagitta error and irregularity.

### A2.3 Rotationally symmetric irregularity

The estimation of this deviation by visual methods is difficult if large amounts of other types of surface form deviation are present. For this reason, digital methods of interferogram analysis are preferred.

If no tilt is present between the surface under test and the reference surface, the fringes appear as concentric circles, but their radii do not increase with the square root of the fringe number, as would be the case with sagitta error. Visual observation of this property is difficult and becomes inaccurate for small deviations. Therefore, the assessment of this type of surface form deviation is practical only in the presence of tilt.

In the presence of tilt, the fringes are W- or M-shaped, depending on the direction of the tilt. In testing the surface, the distance between the surface under test and the effective reference surface should be adjusted so that the apparent sagitta error is zero. This is approximately the case when the two ends and the centre of the fringe nearest the centre of the fringe pattern can be joined by a straight line, as in fig. A2.6. In this case, deviations of the surface from a



sphere are indicated in the fringe pattern by deviations of the fringes from straight lines.2) The rotationally symmetric irregularity is equal to the deviation  $h$  of the fringe from straightness, divided by the fringe spacing  $s$ .

$$\text{Rotationally symmetric irregularity} = \frac{h}{s} \quad (\text{A2.9})$$

In fig. A2.6, the deviation  $h$  of the central fringe from straightness is half the fringe spacing, so the rotationally symmetric irregularity is 0,5 fringe spacings.

If it is not possible to adjust for minimum sagitta error - for instance when using test glasses - then the central fringe shall be compared not to a straight line, but to the circular arc joining the two ends and the centre of the central fringe.

The degree to which the surface form deviation is rotationally symmetric is observed by repeating the above test with the tilt adjusted so that the fringes are oriented in another direction. The surface form deviation is rotationally symmetric if the appearance of the fringes is the same for all orientations of the fringes. The rotationally symmetric irregularity is that part of the deviation which remains the same for all orientations of the fringes.

References: [A2.1]: Malacara, D. ed., Optical shop testing; Wiley, New York, 1978.



## ANNEX 3

## Physical relevance of the RMS surface form deviation

This annex is informative in nature; it is intended as an aid to understanding the rms measures of the surface form deviation defined in part 5 of ISO/DIS 10110.

The peak-to-valley (PV) measures of surface form deviation provided in this part of ISO/DIS 10110 are adequate for describing the surface form deviations of most optical surfaces. However, since these represent only the maximum of deviation of the surface, the PV quantities do not reflect in any way the fraction of the surface which is close to (or far from) the theoretical desired surface. This may be of importance in cases in which the surface form deviation is spatially localised.

The rms measures of surface form deviation defined in part 5 of ISO/DIS 10110 are affected not only by the maximum deviation of the surface, but also by the amount of the surface which deviates from the ideal. For this reason, they can be useful in describing the quality of surfaces, particularly when the degree of spatial localization of the surface form deviation is not known a priori.

The rms deviation of the wavefront transmitted or reflected by any given surface can be related in a simple manner to the rms surface form deviation of that surface. (This requires consideration of the change in refractive index at the surface, as well as the diameter of the beam as it passes through the surface.) The optical quality of an optical system is closely related to the rms wavefront deviation of a beam passing through the system. This relationship, and the manner in which the rms wavefront deviation accumulates as the wavefront passes through the system, are discussed below.

One useful measure of optical quality of a system is the "Strehl definition", or "Strehl ratio", which is defined as the ratio of the intensity at the centre of the image of a point, to that which would be achieved by a perfect optical system. By the central ordinate theorem<sup>1)</sup>, the Strehl definition is also equal to the total volume under the two-dimensional MTF function of the optical system in question. It has been shown<sup>2)</sup> that for systems having small amounts of wave aberrations the Strehl definition is given approximately by

$$S = (1 - 2 \pi^2 \sigma^2)^2,$$

1) The central ordinate theorem states that the value of a function at its origin is equal to the volume under its two-dimensional Fourier transform.

2) Marechal, A. (1947) Rev. d'Optique

where  $\sigma$  is the rms deviation (in wavelengths) of the wavefront from the ideal.<sup>3)</sup> In practice, a Strehl ratio of 80% corresponds to an rms wavefront deviation of 0.07 wavelengths. For the case of sagitta error of the surface, this corresponds to the familiar criterion that the PV deviation not exceed one-quarter wavelength; however, it should be noted that for the reasons given above, the rms criterion is valid for more complex forms of error than is the quarter-wave criterion.

- \* The manner in which the wavefront deviation of an optical system is related to the wavefront deviations contributed by the individual surfaces depends on the extent to which the contributions are correlated. For this reason, little can be said in general about the manner in which the surface contributions combine to form the total system error; nevertheless, it is useful to examine the two extreme cases given below.

If the surface form deviation described by a given type (e.g. irregularity) has the same shape and orientation for all the surfaces in the system, then the PV value of that error type is equal to the algebraic sum (that is, taking the signs into account) of the PV values of the individual contributions of that type. This is always the case for sagitta error, although the signs of the surface contributions are usually not known in advance. (For this reason, some assumptions are necessary when computing the tolerance, even when the form of the error is known.) Surface form errors described by the error type "irregularity" may have different orientations or even different forms for the various surfaces in the system.

If the individual surfaces of the optical system contribute wavefront deviations which are mathematically orthogonal to one another, then the rms wavefront deviation for the system is given by the square root of the sum of the squares of the individual rms values. In most cases, it cannot be expected that the contributions of the surfaces are mutually orthogonal; nevertheless, this may be a useful approximation if the contributions of the surfaces are not expected to be correlated in any way (for instance, when considering the residual aberrations after the removal of sagitta error:  $RMS_1$ ).

Practical cases generally fall between the two extreme cases described above.

3) More precisely,  $\sigma$  is the variance of the wavefront, not its rms deviation; however, there is negligible difference between the two, since the least-squares procedures defined in part 5 effectively remove any constant value of the wavefront before the calculation of the rms quantities.